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REVIEW ARTICLE

Review of the Definitions of Propositions and Statement in their Relationship with Mathematics

Revisión de las definiciones de proposición y enunciado en su relación con las matemáticas
Revisão das definições de proposição e enunciado em sua relação com as matemáticas

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ABSTRACT. In mathematics we work with propositions. The term proposition is taken from logic and is usually defined as a statement that can be qualified as true or false. A proposition is considered as a formulation, and this is a phrase or sentence. According to the above, for example, the expression 3 < 7, which is clearly a true statement from the field of mathematics, would not be such according to what was expressed above. This article reviews the definitions of logic, proposition and statement taken in a sample of high school and college texts that deal with the subject and, on that basis, the consistency of the definitions of proposition and statement are analyzed, as well as their relevance in relation to mathematics.

Keywords: statement, proposition, logic, mathematics, high school and college text

RESUMEN. En matemáticas trabajamos con proposiciones. El término proposición es tomado de la lógica y suele ser definido como un enunciado que puede ser calificado de verdadero o falso. Se considera la proposición como un enunciado y este último como una frase u oración. Según lo anterior, por ejemplo, la expresión 3 < 7, que es claramente una proposición verdadera del campo matemático, no lo sería si atendemos a lo dicho líneas arriba. En el presente artículo se hace una revisión de las definiciones de lógica, proposición y enunciado tomadas en una muestra de textos de secundaria y universitarios que tratan el tema y, a partir de ello, se analiza la coherencia de las definiciones de proposición y enunciado, así como su pertinencia en relación con las matemáticas.

Palabras clave: enunciado, proposición, lógica, matemáticas, textos escolares y universitarios

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RESUMO. Nas matemáticas trabalhamos com proposições. O termo proposição vem da lógica e costuma ser definido como um enunciado que pode ser qualificado de verdadeiro ou falso. Considera-se a proposição como um enunciado e este como uma frase ou oração. Segundo o parágrafo anterior, por exemplo, a expressão 3 < 7, que é claramente uma proposição verdadeira na área das matemáticas, não seria tal se atendêssemos ao mencionado acima. Neste artigo, faz-se uma revisão das definições de lógica, proposição e enunciado, extraídas de uma amostra de textos do ensino médio e superior que tratam o tema e, a partir daí, analisa-se a coerência das definições de proposição e enunciado, assim como a sua pertinência com relação às matemáticas.

Palavras-chave: enunciado, proposição, lógica, matemáticas, textos do ensino médio e superior

Mathematics is a formal science. Taking axioms and definitions as a starting point, a series of propositions are formulated which, expressed in symbolic language and following inference rules, can be validated and build a body of knowledge. Axioms and definitions are conventions. An axiom enunciates in a clear and brief form mathematical truths that, although not always resulting evident, are accepted with no demonstration. According to Young (1929), they are considered as propositions accepted with no demonstration to serve as the basis for the reasoning to be developed. It is clear that in mathematics we work with propositions and this takes us to make their definitions explicit. The definition of a certain term or mathematical concept makes clear the precise meaning attributed to them.

A proposition is usually defined as a statement that can be qualified as true or false. One Pascal's rules for definitions proposes that "when defining, do not employ other class of terms than those clear by themselves or those already defined" (Pascal, as quoted by Young, 1929, p. 174). Since a proposition has been defined as a statement, then the meaning to be ascribed to this last term it is necessary to make the meaning to be ascribed to this last term.

statement is considered as a phrase or sentence. So, following Pascal's rule, we have that either the term "sentence" is clear or we must define it. A review of texts shows that "sentence" is taken as a perfectly clear term. This is like a grammatical structure formed by the union of a subject and a predicate.[1]

The term proposition is taken from Logic. In Logic textbooks, simple propositions are usually expressed as sentences that are either true or false. Associating the term proposition with a sentence is not pertinent when working with mathematical expressions. For example, the expression 3<7 is not a statement and therefore, if we follow what was said above, the expression would not be a proposition. However, 3<7 clearly enunciates a true proposition in the mathematical field. For the above mentioned, it is necessary to review and analyze the definitions of proposition and statement in connection with

1 Diccionario de la Real Dictionary of the Real Academia Española. Definition taken from http://dle.rae.es/?id=R8LGdYW on 18/Dec/2016.

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Mathematics. Since the construction of the body of mathematical knowledge is done formulating propositions, then the definition of the term proposition must be pertinent to Mathematics. Herein the importance of this study. In this work, a review is done of the definitions of Logic, proposition, and statement taken from a sample of high-school and college textbooks which address the topic.

LOGIC AND MATHEMATICS

"Logic and Mathematics, occupied with inventing formal entities and establishing relationships among them, are often called formal sciences" (Bunge, 1966, p. 7). They are deductive sciences in which demonstrations are based on a set of definitions and their own rules. Due to being formal sciences, Mathematics and Logic maintain a close relationship. Concepts and principles subject of Logic examine the mechanisms of rational thinking, thus its comprehension facilitates the learning of Mathematics. But not only of Mathematics. The teaching of Logic "enables the adequate handling of the language of science and as a method to build science" (Vásquez, 2013, p. 155). Mathematics and Logic are considered important areas in education. However, while the study of mathematical topics has been maintained in school programs, Logic topics have been diluting.

Currently, in official documents of regular basic education, the study of Logic does not appear as an independent area. Even the National Curriculum Design (DCN, as per its Spanish acronym) of 2009, which ruled for several years after, included in the area of Mathematics, knowledge of logical relationships and sets. In the third version of the national curriculum system framework (MCN, in Spanish), in the description of the competency acts mathematically in diverse contexts, it is stated that Mathematics helps face and assume the problems that the world presents to us in a reasoned and logical manner. Logical and critical reasoning is highlighted in that it "favors the logical coherence of ideas... as well as how to make deductions and inferences" (MINEDU, 2014, p. 74). Everything indicates that only with the learning of Mathematics this reasoning would develop. They are not few the voices that, in their moment, were raised for removing the subject of Philosophy, and with it Logic, from the official basic education program. Among the criticism, there was a warning that the curriculum design did not include Logic, which allows the students to exercise rigorous thinking.[2].

But there were also voices against. Boisvert (2004) indicates that "it is true that Logic is useful for the training in and the exercise of critical thinking, but it is not the only existing tool" (p. 62). In 1948 basic education included the study of Logic. That year, Miró Quesada, renowned Peruvian investigator who concerned himself with introducing Logic in Peru, published Logic school subject, where he states that his textbook adjusts to the official program. Later, in 1961, the topics of Logic became part of the subject of Philosophy. There it stayed until 2002, year in which it was removed from school programs. In higher education, despite the importance it has for scientific research, interest for Logic has also been weakened. In his studies about Logic textbooks, Vásquez (2013) refers that "the teaching-learning of general Logic at the university level, from 1940 to now, shows a lessening of its importance in professional education" (p. 153).

2 In 2004, two years after the governmental measure of removing the subject of Philosophy, the Peruvian philosophical community declared openly in favor of reestablishing it in schools.

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In basic education, within the topics of numbers, relationships and functions in the area of Mathematics, knowledge of logical relationships and sets was included. These, in turn, included all that referred to statement and proposition. Its study started in secondary school, in second grade and continued until fifth grade. That is why this topic is studied by all highly demanded textbooks published by the main publishers in our country. In higher education the topics of Logic are also usually included in the first Mathematics course in general studies or cross-sectional formation. To include Logic topics in the syllabus of a Mathematics course assumes that the contents addressed there will contribute to the learning of Mathematics. It would be expected that concepts, rules and examples be related to topics native to Mathematics. More reason if the topics of Logic are the first to be studied in the course. In general, after the chapter about Logic, follows the one that addresses topics of sets. This is justified since the theory of sets can be built using logic connectors and quantifiers. For the following mathematical topics, it would be justified since many theorems are enunciated under the form "If..., then..." Its demonstration requires Logic for a solid argumentation.

For this work, an intentional sample of Mathematics and Logic textbooks was taken in which topics of Logic are addressed. The sample included seven high-school textbooks from the main Peruvian publishers and nine college-level textbooks. The review of these textbooks focused on the following aspects:

a) Analysis of the definitions of Logic, proposition, and statement.
b) Coherence of the definition of statement concerning the definition of proposition.
c) Coherence of the definitions of statement and proposition concerning the definition of Mathematics.

DEFINITION OF LOGIC IN TEXTBOOKS

Logic is usually the first topic studied in Mathematics textbooks from second to fifth grade in secondary school. In textbooks of the first Mathematics course in higher education, the topics of Logic are usually presented in the first chapter. We find different definitions or descriptions to refer to it.

Definitions taken from high-school textbooks:

L1. The discipline that determines through rules and techniques the validity of an argument is called Logic (Grupo Editorial Norma, 2009a, p. 12)
L2. Logic is the discipline that helps determine if the reasoning is valid or not. Starts off a set of information to deduce another. The initial information is called the premise, and for the analysis, it is considered true, the one deduced is the conclusion (Grupo Editorial Norma, 2009b, p. 12).
L3. The science that studies the formal principles of knowledge, that is, those conditions that must be fulfilled for an statement, whatever its contents may be, to be considered true, and not only a hypothesis, is called Logic (Grupo Editorial Norma, 2009c, p. 12).
L4. Logic is the science that allows us to learn to reason correctly (Doroteo & Gálvez, 2005, p. 12)
L5. Logic is the science that studies the methods and procedures that determine the validity or invalidity of inferences or arguments. To infer is reasoning to obtain a conclusion from a set of statements that we call premises (Editorial Santillana, 2009, p. 18).

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Definitions taken from college textbooks:

L6. The science that studies the methods or procedures that apply definitions and laws or rules with the purpose of determining the validity or invalidity of inferences (Rosales, 1994, p. 15).

L7. Logic is the study of methods and principles used to distinguish the correct reasoning from the incorrect one (Copi, 1964, p. 17).

L8. Logic is the science and at the same time the art of the study of reasoning or inferences (Katayama, 2003, p. 19).

From the definitions above we can observe that Logic is considered a discipline (L1, L2), a science (L3, L4, L5, L6, L8), and an art (L8). Logic and Mathematics are considered formal sciences. Palacios and García (1998) point out that science can be defined as a system of propositions and that the formal sciences "establish the truth of its propositions by means of deductions or demonstrations" (p. 61). Miró Quesada (1978) defines Logic as "the theory of deduction" (p. 129)[3]. It is a scientific theory since it comprises a set of knowledge systematically linked that deals with a well-defined field of study which, in the case of Logic, is deduction. Miró Quesada points out that, among other things, when there is a method that allows justifying and augmenting knowledge with safety, it is a scientific theory. On L6, Rosales seems to want to highlight this characteristic when saying that Logic, with the purpose of determining the validity or invalidity of an inference, studies the methods or procedures that apply to the definitions and rules given. The same seems to happen in L7 when Copi (1964) refers to Logic as the study of methods and principles.

Although the definition L7 does not indicate it, Vásquez (2013) declares that in his Introduction to Logic, Copi explicitly states as one of the objectives "to study Logic as an art and as a science; for this, it must exercise permanently" (p. 162)[4]. In L8 Katayama (2003) defines Logic as science and art but does not explain why he considers it this way[5]. We can find in Aristotle references about considering Logic as an art. For Aristotle, considered the father of Logic, art and science are different forms of knowledge. The first one is technical knowledge whose purpose is productive while the second one does not have any specific purpose and constitutes theoretical knowledge that studies causes and principles of things. In his Metaphysics, Aristotle (2007) says that, as opposed to other animals, "mankind has to handle itself, art, and reasoning" (p. 7). Saint Thomas (as quoted in Moreno, 1971) in his Commentary on the Second Analytics writes "it is necessary then an art that directs the act of reason itself (...) This art is Logic, that is, rational science" (p. 4). Logic was interpreted as the art of arts since it would the one directing the act of reason.

The review of the eight textbooks mentioned has made evident that there is no univocity in the definition of Logic and the description of the object of study. This can be explained because one definition is not definitive. It is a convention of the meaning attributed during the time it remains current. The extension and sense of the term are limited by the scope of the group to which the definition is directed. According to Young (1929) "the scope of the point of view of the student is what should control the definition, not the teacher's" (p. 178). As the student's horizon widens, it is reasonable for definitions to be expanded when advancing in secondary school and, above all, when passing to the university stage.

3 The definition is taken from the part corresponding to Logic written by Francisco Miró Quesada from the high-school textbook Introduction to Philosophy and Logic. The textbook fusions the books Introduction to Philosophy by Augusto Salazar Bondy (1925-1974) and Logic by Miró Quesada. The latter, published after Salazar's death, readjusts the topics and introduces other to adapt them to the official program current in 1978.

4 The definition of Copi is frequently quoted. Vásquez states that his book has been the basis for training many teachers who teach Logic at the university level now.

5 In the introduction of his book, Katayama points out that the text has as an objective a panoramic presentation of the main applications of Logic.

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However, our review shows that the definitions given in high-school textbooks do not advance towards a generalization. At the same time, textbooks from different grades of the same publisher do not adopt the same definition of Logic, being the one given in the lower grade (L1) more general than the one in higher grade (L3). The description of the object of study of Logic refers to terms such as inference, argument, and reasoning. In general, these terms are not defined, or no notion is given about them.

REASONING, INFERENCE, AND ARGUMENT

EIn the definitions quoted we see that, when describing the object of study of Logic, different terms are alluded such as reasoning (L2, L4, L7, L8), inference (L5, L6, L8), argument (L1, L5), and statement (L3). But in the textbooks from which these definitions are taken, the meanings of the terms that those definitions refer to are not always precise. In L8, Katayama (2003) takes inference and reasoning as synonym terms. In his book, he does not make any distinction between the two and neither defines them. However, describes argument as a type of reasoning or inference characterized for upholding a conclusion supported by premises. L7 is also quoted, together with definitions by Moreno (1971) in his *Mathematical Logic*[6]. He does not specify the meaning of the term reasoning but indicates that Logic "investigates questions linked to the validity or invalidity of the argumentation" (Moreno, 1971, p. 3). He neither defines the argument but initially mentions that the central problem of Logic is to establish the conditions by which "an statement can be considered as a conclusion derived from other statements called premises" (Moreno, 1971, p. 1). In another text of the same author, we find that "all systems of premises and conclusion are called argumentation" (Moreno, 1967, p. 13). This same definition is adopted by Bustamante (2009) who, to obtain the purpose of distinguishing correct reasoning from incorrect reasoning, starts by considering the elements of language used to express this reasoning. In definition L6, Rosales talks of inferences y defines inference as "a process that consists of obtaining the conclusion from a set of premises" (Rosales, 1994, p. 15)[7].

As previously mentioned, for Miró Quesada (1978) Logic is the theory of deduction which consists of passing from the truth of the premises to the truth of the conclusion. He talks about deduction and not of inferences or reasoning. Makes the distinction between them but indicates that he will use the terms deduction and inference as synonyms. Distinguishes between deduction and inference because the latter can also mean induction, which is a kind of knowledge that is different from deduction that belongs to Logic. Distinguishes between deduction and reasoning since this would be any kind of thinking that tries to justify a thesis rationally. In this way, he considers that a justification by analogy, an inductive inference, an experimental verification, or the invalidation of a hypothesis by a counter example is also reasoning.

DEFINITIONS OF PROPOSITION IN TEXTBOOKS

In the definition of Logic, reference is also made to terms such as reasoning, argument, inference, and deduction. The review shows that they are given the same sense when the purpose of Logic is discussed. We also see that all of them talk about premises and conclusion. For Bustamante (2009)

6 This textbook collects the notes of the course of Logic given in 1967 at the School of Philosophy and Arts of Universidad Nacional de Buenos Aires.

7 Vásquez points out that Rosales's textbook Introduction to Logic is, perhaps, the most used in our country's universities, emphasizing that its effectiveness has been demonstrated on the new generations of Logic teachers and on professionals who received Logic topics during their training.

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"an argument or reasoning is a block of propositions with which it is stated that one of them, called conclusion, is derived, deduced or followed as a consequence of other propositions of the same block called premises" (p. 11). Most authors consider the premise and conclusion as propositions. Next, we will mention different definitions or descriptions for proposition.

Definitions in high-school textbooks:

P1. Has an assertive character, because it affirms or rejects something about a known subject. One expression can be verified or rejected which gives it a value of truth by which it can be true or false but not both (Grupo Editorial Norma, 2009a, p. 12).

P2. Every coherent statement characterized for having a truth value, true of false with no ambiguity in a particular context, is called a proposition (Grupo Editorial Norma, 2009c, p. 12).

P3. They are statements whose meaning can be qualified as true or false. Formally it is said that a proposition has a unique value of truth: true or false but not both (Grupo Editorial Norma, 2009d, p. 12).

P4. A logical proposition is every statement that has a truth value: true (T) or false (F) but not both at the same time (Doroteo & Gálvez, 2005, p. 12).

P5. Propositions are language expressions that can be classified as true or false. Questions, commands or exclamations are not propositions (Editorial Santillana, 2009, p. 18).

P6. It is a statement that has a truth value (true or false) (Editorial Santillana, 2012, p. 11).

P7. A proposition is the expression of everyday language that admits the possibility of being true or false, but cannot be true and false at the same time (Coveñas, 2012, p. 9).

Definitions in university textbooks:

P8. A proposition is a statement that can be qualified either true or false but not both at the same time (Curo & Martínez, 2013, p. 13).

P9. A proposition is a statement that can be true or false (Zúñiga, 2013, p. 1).

P10. Propositions are language expressions for which makes sense to say they are true or false (Rosales, 1994, p. 16).

P11. They are expressions or sentences that can be qualified as true (T) or false (F) but not both simultaneously (Chávez, 1997, p. 1).

Other definitions:

P8. A proposition is defined as an assertion that can be true or false but not both (Miller, Horeen & Hornsby, 2006, p. 95).

P13. A proposition is a declarative phrase that can be affirmed or rejected (Bustamante, 2009, p. 7).

P14. They are those expressions or sentences that can be qualified either as true or false, with no ambiguities (Venero, 2012, p. 1).

The definition of argument (reasoning, deduction or inference) makes reference to premise and conclusion, and these are defined as propositions. That the knowledge brought is clear and precise and the relationship between them is well established is another of the things that Miró Quesada (1978) refers to as a scientific theory. To define is to set with clarity, accuracy, and precision the meaning of a word or the nature of one person or thing[8]. The definition provides a formal description of something. "The

8 Dictionary of the Real Academia Española

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notion of a described thing should exist with clarity in the mind before that description is established" (Young, 1929, p. 174). Sentence and phrase are language expressions studied at a very early age. Both comprise a set of words with sense, with the sentence differentiating itself by expressing complete grammatical sense. For Miró Quesada (1978) "from the point of view of Logic, a proposition is a sentence or phrase susceptible to be true or false" (p. 127). Phrases are expressions such as "What time is it?" (interrogative), "Long live Peru!" (exclamatory), and "Stop yelling!" (imperative). However, none of them is a proposition since they cannot be qualified as true or false. On the contrary, the sentence, "Lima is the capital of Peru," is a proposition because it states a truth.

Of the 14 definitions of propositions presented we have that six of them referred to propositions as statements (P2, P3, P4, P6, P8, and P9). Of these, four were taken from school textbooks (P2, P3, P4, and P6). Observe that definitions P2 and P3 correspond to 4th and 5th grade of high-school textbooks from the same publisher. While these definitions do not contradict each other, we think that it is not necessary to change its description from one grade to the other. We could say the same with respect to L1, L2, and L3. There are objects whose definitions can be expanded as one advances grades. Enough mentioning the expansion of number from natural to real. But we consider that in this case is not justified. Definition P1, taken from a textbook from the same publisher of P2 and P3, is the most confusing event though it is directed to 2nd-grade students. In it, it is no specified if a proposition is a statement, expression, sentence, phrase or other. It is mentioned that it has an assertive character because it affirms or rejects something about a known subject. It does not make explicit what it is referring to as "subject," although it proposes as an example $x<397$, pointing out that this is not a proposition because the subject is unknown. Here the subject would be the variable x. It seems to suggest that since x is not known, it cannot be stated the truthfulness or falsehood of the given expression. However, the reason is not that x is not known but that nothing is known about it. Thus the expression "If $x>1$, then $x2>1$" is a true proposition even though we do not know the value of x. In the 3rd-grade textbook of this same publisher, looking to complement the definition of Logic, it mentions that it "analyzes the structure of the affirmations and expresses them symbolically. For this, it considers as elements of language propositions, propositional functions, logical operators and quantifiers" (Grupo Editorial Norma, 2009b, p. 12). It is not defined what a proposition is. Maybe expecting this definition to be taken form the textbook from the previous grade which, as pointed out, in our opinion is more imprecise.

Of the other eight definitions, we find that five of them referred to propositions as expressions (P5, P7, P10, P11, P14) and of these, there are two that also consider them as sentences (P11, P14). One definition refers to propositions as an assertion (P12) and another, as a declarative phrase (P13). Here we find that P5 and P6, corresponding to second and fourth-grade textbooks of the same publisher (different from the former) define a proposition referring to different terms. In one case it refers to an expression and in the other to a statement. We cannot say that these definitions contradict themselves, but we neither consider pertinent to change their description. One definition stands out over the other due to its simplicity and precision. We saw that the definitions provided by college textbooks are briefer (16 words on average) and more precise than those in school textbooks (26 words on average).

The definitions reviewed show that, although in different number and in different words, a greater uniformity exists in the definition of the term proposition. In them, the main characteristic of the term

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in its relationship with Logic and Mathematics is pointed out; this is the fact of being a statement or expression of the language that may be qualified as true or false. At the same time, we find that the definitions for proposition on higher education textbooks show a greater level of generalization with regards to those given in high-school textbooks.

DEFINITION OF STATEMENT IN TEXT BOOKS.

Defining a new term supposes making use of other terms whose meanings are clear. The definition P9 "a proposition is a statement that can be true or false," perhaps for its briefness is usually mentioned by Mathematics teachers (and students) when they are asked about the meaning of proposition. P9 considers the proposition as a statement. We find this in definitions P2, P3, P4, P6 and P8 that also refer to statement. Because of this it is justified to review and specify the meaning of statement. Definition P9, taken from a university textbook, defines proposition as a statement, but in the book it is not specified what is understood as statement. The same occurs with the 5th-grade textbook from which P3 was taken. Of the six definitions that refer to proposition as statement, four of them define the term statement. The contrary occurs in definition P1 in which Chávez (1997) does not refer to proposition as statement. However, in E5 he provides, in a simple form, the notion of this term.

E1. Every phrase or sentence is called statement. Statements that are commands, desires, interrogations or exclamations are propositions (Grupo Editorial Norma, 2009c, p. 12).
E2. A statement is every phrase or sentence utilized in common language. Some statements are affirmations, commands, interrogations, exclamations, etc. (Curo & Martínez, 2013, p. 13).
E3. A statement is every phrase o sentence. Some statements are commands, exclamations and interrogations; others are affirmations or negations (Editorial Santillana, 2012, p. 11).
E4. A statement is every language expression (Doroteo & Gálvez, 2005, p. 12).
E5. Enunciation: Sentence or phrase (Chávez, 1997, p. 1).

Notice that E1, E2, E3, and E5 describe a statement as a phrase or sentence. This is usually the most common description for a statement. Moreno considers the proposition as the meaning of a statement. "With the word statement, we mean a linguistic expression that by tending towards a reality acquires the characteristic of being true or false" (Moreno, 1971, p. 1). For Moreno, due to their linguistic nature, statements are material things easier to manage, which makes it easier to learn its structure than a proposition's. Bunge (1980) refers to nominalists who deny that there are propositions and prefer to talk of sentences. Sentences, by belonging to written or spoken languages, have a concrete existence. "Almost anybody can pronounce or write a sentence in some language but never could somebody see or hear a proposition" (Bunge, 1980, p. 65). Because propositions are not concrete objects, nominalists do not see the need for propositions and are satisfied with sentences. Miró Quesada (1978) points out, about the distinction made between statement and proposition, that "most authors consider that a proposition and a statement are the same: expressions that describe facts" (Miró Quesada, 1978, p. 132). Bunge also says that just like Arithmetic authorizes us to affirm the existence of integer number, we can admit the existence of conceptual objects like propositions. This is consistent with the character of formal science that pure Logic has. Formal sciences "occupy themselves with the forms and relationships of thought and of entities constituted by thinking, doing

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without the real facts to which they may be referring" (Rodríguez, 2002, p. 66). Making the distinction between a proposition and the sentences that enunciate it, Bunge (1980) says that "the significance of a sentence is a proposition" (p. 69) where the meaning of a proposition is made up of its sense and its referents. A proposition out of context lacks a precise meaning. It is the explicit mention of the context which allows tracking the logical links of a proposition and determines its full sense.

We observe that definitions E1, E2, E3, and E5 restrict the propositions to the natural language. According to this definition, mathematical expressions such as 5>2 or sen(30°)=0.8 would not be propositions because they are not phrases or sentences. However, they can be perfectly qualified as true and false respectively. That is why it is required to expand the field of statement so that it includes expressions of this type. In E4 we find a more open definition when mentioning "every expression of language," although we consider it would have been better to say "every expression of one language." Just like natural language, the mathematical language has its own syntax and semantics. These allow determining if a mathematical expression makes sense and, from there, determining its truth value. In P11, Chávez (1997) seems not wanting to restrict propositions to the linguistic field since he defines them as "expressions or sentences" and by giving as examples two mathematical expressions "7-1=2+9-5" and "2+3>5", propositions to which he assigns truth values of T and F respectively. The same could be understood from the definition given by Venero (2012) who, in his definition P14 of proposition, tries to distinguish between "expressions" and "sentences" when giving as an example of a true proposition 17–6=11. In the analysis of an expression, as far as it can be considered as a proposition or not, it is important to have a precise context that brings sense and reference to the expression under analysis.

If we define proposition (P) in terms of statement (E) and this as an expression in a given language (L), the essential is that E has meaning in L, so that it can be said that P is either true or false. For Russell, "just an understanding of language is necessary to know a logical proposition" (as quoted in Monk, 2013, p. 78). We can say that a statement is an expression, in natural or mathematical language, about a question with its own sense. So understood as a statement, the expressions "2+1=5" and "log (1) =0" are propositions while "x+2" and "7<" are not because they make no sense. The assertion that the expression "2+1=5" is a proposition can be justified by enunciating it in natural language "two plus one is equal to five." This last is a phrase with a full sense that declares something evidently false. The former shows an example of different statements that have the same meaning. Thus, the statements "*tres es mayor que dos*," "three is greater than two," and "3>2" have the same meaning. In this line, there are some who consider a proposition as the common content of synonym declarative sentences. Understanding language as a system of signs used by a community to communicate orally or in writing, the definition of proposition given by Piscoya (2007) is pertinent: "is every finite sequence of signs which with sense may be qualified as true or false" (p. 32). This general form of defining a proposition can also be found in Wikipedia "a proposition is a chain of signs expressed in a particular language," also indicating that what makes this chain a proposition is that it is interpretable. Bunge (1980) admits that propositions lack autonomous existence and exist only conceptually or formally, but adds that if we want to do mathematical logic, we must pretend that they exist. To affirm the conceptual existence of a proposition takes us to think or pretend that they are thinkable in a rational manner. We know the importance that this has for Mathematics whose nature is, essentially, deductive.

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CONCLUSION

Mathematics is a science that works with ideal entities. Whether they are abstract or interpretations of reality, these formal entities required to be defined for their study. The body of mathematical knowledge is built deductively from axioms and definitions. In Euclidean Geometry, we have one of the closest examples of deductive construction. Each textbook or Mathematics course is in itself a body of knowledge that must be constructed. In the same branch of Mathematics, contents are arranged sequentially so that one topic is the expansion of the prior. In turn, terms and concepts of a topic serve as a starting point to define the terms and concepts of the next one. Herein the importance of definitions in that they give meaning to terms and objects with which we work in Mathematics.

Definitions allow to describe the properties of a mathematical object and, within the corresponding theoretical framework, the combination of its properties allow to enunciate a set of mathematical propositions. These propositions can be represented as properties, lemmas, theorems or corollaries. Many theorems are enunciated in the form "If p, then *q*," that is, as a conditional proposition. As opposed to axioms, theorems must be demonstrated. A demonstration is reasoning through which the truth of a proposition is established. Essentially, to demonstrate means to prove. So demonstrating the truthfulness of a mathematical proposition means to prove, using reasoning, that what the mathematical proposition says is true. This assumes the use of definitions and other related propositions presented previously. But it also assumes making explicit a set of steps that, duly justified, allow showing the truthfulness of a mathematical proposition. What is essential in the demonstration is the way how we justify the steps we follow to demonstrate. Different taxonomies in the domain of Mathematics consider the cognitive activity of demonstrating as belonging to higher levels.

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